Calc BC Parametric Practice Name: Key Block: Seat:

1. 2001 BC 1

An object moving along a curve in the xy-plane has position $(x(t), y(t))$ at time t with

$$\frac{dx}{dt} = \cos\left(t^3\right) \text{ and } \frac{dy}{dt} = 3\sin\left(t^2\right)$$

for $0 \le t \le 3$. At time $t = 2$, the object is at position (4,5).

(a) Write an equation for the line tangent to the curve at (4,5). (1 pt)

(b) Find the speed of the object at time $t = 2$. (1 pt)

(c) Find the total distance traveled by the object over the time interval $0 \le t \le 1$. (3 pts)

(d) Find the position of the object at time $t = 3$. (4 pts)

a) $\frac{dy}{dx} = \left.\frac{\frac{dy}{dt}}{\frac{dx}{dt}}\right|_{t=2} = \frac{3\sin 4}{\cos 8} = 15.60417154$

$y - 5 = 15.604(x-4)$

b) $\sqrt{(\cos 8)^2 + (3\sin 4)^2} = 2.2750649$

c) $\int_0^1 \sqrt{\cos^2(t^3) + 9\sin^2(t^2)}\, dt = 1.458322975$

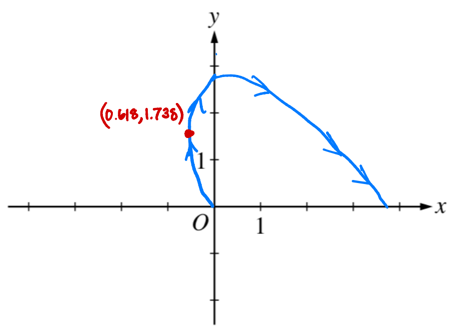
d) $x(3) = 4 + \int_2^3 \cos t^3 \, dt = 3.953501901$

$y(3) = 5 + \int_2^3 3\sin t^2 \, dt = 4.906358113$

2. 1999 BC 1

1. A particle moves in the xy-plane so that its position at any time t, $0 \le t \le \pi$, is given by $x(t) = \frac{t^2}{2} - \ln(1+t)$ and $y(t) = 3\sin t$.

(a) Sketch the path of the particle in the xy-plane below. Indicate the direction of motion along the path. (2 pts)



(b) At what time t, $0 \le t \le \pi$, does $x(t)$ attain its minimum value? What is the position $(x(t),\ y(t))$ of the particle at this time? (3 pts)

(c) At what time t, $0 < t < \pi$, is the particle on the y-axis? Find the speed and the acceleration vector of the particle at this time. (4 pts)

b) $x'(t) = t - \frac{1}{1+t} = 0$

$t(1+t) - 1 = 0$

$t + t^2 - 1 = 0$

$t^2 + t + \frac{1}{4} = 1 + \frac{1}{4}$

$(t + \frac{1}{2})^2 = \frac{5}{4}$

$t + \frac{1}{2} = \frac{\sqrt{5}}{2}$

$t = -\frac{1}{2} + \frac{\sqrt{5}}{2} \approx 0.618$

$y(.618) = 1.738$

$y(\phi) = 1.738$

(c) $x(t) = \frac{t^2}{2} - \ln(1+t) = 0$ ✓

at $t = 1.285$ (or $t = 1.286$) ✓

$x'(t) = t - \frac{1}{1+t}$ $y'(t) = 3\cos t$

Speed $= \sqrt{[x'(1.285)]^2 + [y'(1.285)]^2} \approx 1.196$

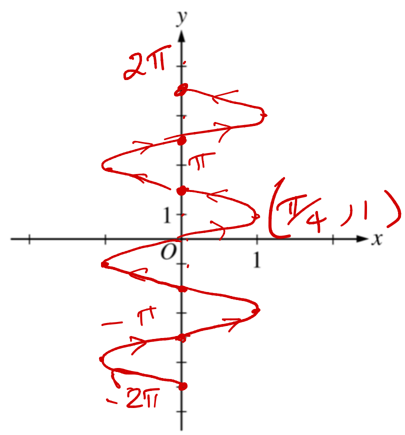
acceleration $\langle x''(1.286),\ y''(1.286) \rangle$

$\langle 1.191,\ -2.879 \rangle$

3. 2002 BC 1 (Form B)

1. A particle moves in the xy-plane so that its position at any time t, for $-\pi \le t \le \pi$, is given by $x(t) = \sin(3t)$ and $y(t) = 2t$.

   (a) Sketch the path of the particle in the xy-plane provided. Indicate the direction of motion along the path. **(Note: Use the axes provided in the test booklet.)** (2 pts)



   (b) Find the range of $x(t)$ and the range of $y(t)$. (2 pts)

   (c) Find the smallest positive value of t for which the x-coordinate of the particle is a local maximum. What is the speed of the particle at this time? (3 pts)

   (d) Is the distance traveled by the particle from $t = -\pi$ to $t = \pi$ greater than 5π? Justify your answer. (2 pts)

(b) $x \in [-1, 1]$
$y \in [-2\pi, 2\pi]$

(c) $\frac{dx}{dt} = 3\cos(3t) = 0$

$3t = \frac{\pi}{2}, \frac{3\pi}{2}$, etc

smallest pos

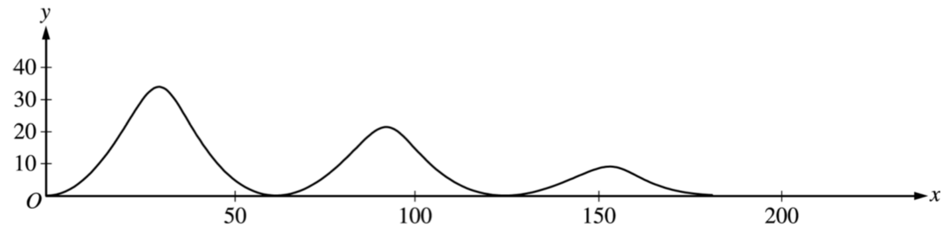
$t = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}$

speed $= \sqrt{\left[x'\left(\frac{\pi}{6}\right)\right]^2 + \left[y'\left(\frac{\pi}{6}\right)\right]^2}$

$= \sqrt{\left(3\cos\frac{\pi}{2}\right)^2 + 2^2} = \sqrt{0+4} = 2$

(d) $\int_{-\pi}^{\pi} \sqrt{(3\cos 3t)^2 + (2)^2}\, dt \approx 17.973 > 5\pi$

4. 2002 BC 3



3. The figure above shows the path traveled by a roller coaster car over the time interval $0 \le t \le 18$ seconds. The position of the car at time t seconds can be modeled parametrically by

$$x(t) = 10t + 4\sin t$$
$$y(t) = (20 - t)(1 - \cos t),$$

where x and y are measured in meters. The derivatives of these functions are given by

$$x'(t) = 10 + 4\cos t$$
$$y'(t) = (20 - t)\sin t + \cos t - 1.$$

(a) Find the slope of the path at time $t = 2$. Show the computations that lead to your answer. (1 pt)

(b) Find the acceleration vector of the car at the time when the car's horizontal position is $x = 140$. (2 pts)

(c) Find the time t at which the car is at its maximum height, and find the speed, in m/sec, of the car at this time. (3 pts)

(d) For $0 < t < 18$, there are two times at which the car is at ground level ($y = 0$). Find these two times and write an expression that gives the average speed, in m/sec, of the car between these two times. Do not evaluate the expression. (3 pts)

(a) $\left.\frac{dy}{dx}\right|_{t=2} = \left.\frac{y'(t)}{x'(t)}\right|_{t=2} = \frac{(20-2)\sin 2 + \cos 2 - 1}{10 + 4\cos 2} = 1.794$ m/sec

(b) $140 = x(t)$
$140 = 10t + 4\sin t$
$140 - 10t - 4\sin 4 = 0$
at $t = 13.64708271 \Rightarrow \boxed{A}$
$\langle x''(A), y''(A) \rangle = \langle -3.5291, 2.9903 \rangle$ (in m/sec^2)

(c) $y'(t) = 0$ for first time $t = 3.02391584 \Rightarrow \boxed{B}$
speed $= \sqrt{[x'(B)]^2 + [y'(B)]^2} \approx 6.0276$ m/sec

(d) $y(t) = 0$ on $(0, 18)$
when $t = 2\pi$ sec. and $t = 4\pi$ seconds
the value is speed, using average value
$\frac{1}{b-a}\int_a^b f(t)\,dt$
$\frac{1}{4\pi - 2\pi}\int_{2\pi}^{4\pi} \sqrt{[x'(t)]^2 + [y'(t)]^2}\,dt$